Analysis of Coplanar Waveguide Radiating End Effects Using the Integral Equation Technique

M'hamed Drissi, *Member, IEEE*, Victor Fouad Hanna, *Senior Member, IEEE*, and Jacques Citerne

Abstract —**An integral equation technique solved by the moment method associated with the single one-port model is used to analyze radiating end effects of coplanar waveguides (CPW's). Theoretical results obtained on a CPW short circuit end are compared with those obtained experimentally using series-gap-coupled straight CPW resonators.**

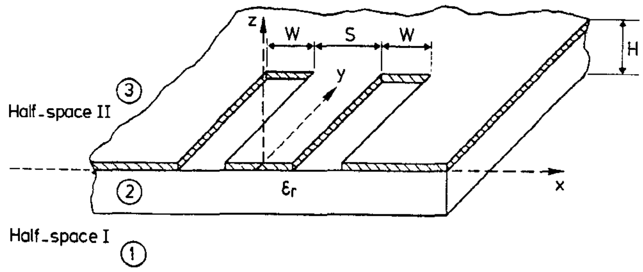

Fig. 1. Pseudo short circuit of a coplanar waveguide.

I. Introduction

The extensive use of either open or short-end coplanar waveguide (CPW) terminations (Fig. 1) as tuning stubs in transitions [1] and in the design of planar balanced mixers [2] and detectors [3] requires design data based on accurate models that account for radiation and discontinuity dispersion effects.

Many efforts have been made to characterize microstrip discontinuities, but only a few have been devoted to determining their CPW counterparts, either theoretically using a full-wave analysis (e.g. [4] and [5]) or experimentally [6]. This is despite the fact that CPW's offer several advantages over conventional microstrip lines for monolithic or hybrid MIC applications because of their easy adaptation to both parallel and series insertion of both active and passive components.

The present paper gives a rigorous analysis to characterize radiating end effects of CPW's using the integral equation technique. This analysis has the advantage of being performed in the space domain, which allows the electric and magnetic field distributions in the whole space to be determined directly without the use of an inverse Fourier transform. Then an experimental method is given for the measurement of the reactance of a CPW short circuit end that allows its radiation resistance to be determined.

II. Analysis

During the analysis presented here, it is assumed that the dielectric is lossy and isotropic and that it is infinite in extent in the xy plane. A further assumption is that the conductors are perfect and have negligible thickness. The transverse electric field components in the slots are taken into consideration in our analysis while the longitudinal fields are neglected. This assumption is valid as long as the slot width is very small compared with the wavelength.

A. Magnetic Integral Equation

As a first step, the equivalence theorem [7] is applied to divide the problem into two separate parts (Fig. 2). The apertures are covered by perfect conductors surrounded on the two sides by equivalent surface magnetic current $+\vec{M}_s$, $-\vec{M}_s$ satisfying the following equation:

$$\vec{M}_s(\vec{r}_t) = \vec{e}_z \times \vec{E}_s(\vec{r}_t) \tag{1}$$

where $\vec{E}_s$ is the electric field in the apertures, $\vec{e}_z$ is the unit vector normal to them, and $\vec{r}_t$ is the position vector between the excitation source and the observer.

The integral equation formulations can then be obtained from the boundary condition for the total tangential magnetic field at the aperture surface. This condition can be written in the following form:

$$\vec{e}_z \times \left[\vec{H}^{\mathrm{I}}(\vec{r}_t) - \vec{H}^{\mathrm{II}}(\vec{r}_t) + \vec{H}^{\mathrm{ex}}(\vec{r}_t)\right] = \vec{0}. \tag{2}$$

Here $\vec{H}^{\mathrm{ex}}$ is the feeding field and $\vec{H}^j$ $(j = \mathrm{I, II})$ is the scattered field, which can be derived from a scalar potential U and a vector potential $\vec{F}$ as

$$\vec{H}^j(\vec{r}_t) = -j\omega\vec{F}^j(\vec{r}_t) - \nabla U^j(\vec{r}_t). \tag{3}$$

These vector and scalar potentials are in turn expressed in term of integral dyadic, $\overline{\overline{G}}_F$, and scalar, G_U, magnetic Green's functions, weighted by the equivalent current $\vec{M}_S$ and the

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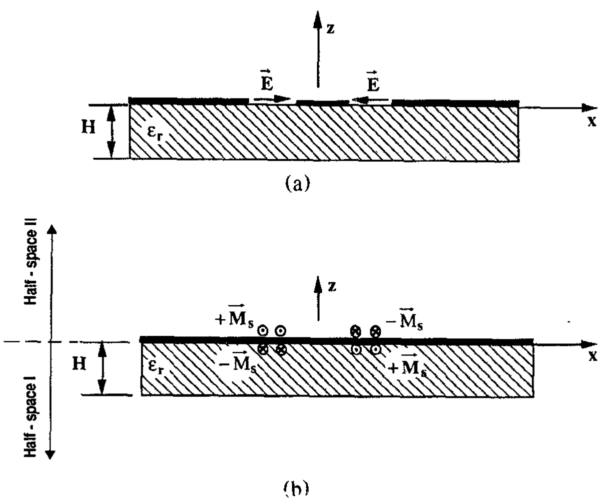

Fig. 2. Transverse cross section of a coplanar waveguide: (a) original problem; (b) equivalent problem.

equivalent charge τ_S distribution respectively. Thus,

$$\vec{F}(\vec{r}_t) = \int\int_{S_f} \bar{\bar{G}}_F(\vec{r}_t/\vec{r}_\nu)\cdot\vec{M}_S(\vec{r}_\nu)\, dS' \tag{4}$$

$$U(\vec{r}_t) = \int\int_{S_f} G_U(\vec{r}_t/\vec{r}_\nu)\tau_S(\vec{r}_\nu)\, dS' \tag{5}$$

where $\vec{r}_\nu$ is the source vector, and S_f is the slot cell surface.

Finally, the magnetic integral equation can be written as

$$\vec{e}_z \times \sum_{\nu=1,2}\left[\int_{S_{f_\nu}} j\omega\left(\bar{\bar{G}}{}^{\rm I}_F(\vec{r}_t/\vec{r}_\nu) + \bar{\bar{G}}{}^{\rm II}_F(\vec{r}_t/\vec{r}_\nu)\right)\cdot\vec{M}_S(\vec{r}_\nu)\, dS + \int_{S_{f_\nu}} \nabla\left(G^{\rm I}_U(\vec{r}_t/\vec{r}_\nu) + G^{\rm II}_U(\vec{r}_t/\vec{r}_\nu)\right)\tau_S(\vec{r}_\nu)\, dS\right] = \vec{e}_z \times \vec{H}^{\rm ex}(\vec{r}_t). \tag{6}$$

It is to be noticed that the surface magnetic charges τ_S are related to the equivalent magnetic currents $\vec{M}_S$ by the known continuity equation

$$\nabla\cdot\vec{M}_S + j\omega\tau_S = 0. \tag{7}$$

The Green's functions of (6) correspond to the potentials created by the equivalent unit moment magnetic sources placed on the ground plane. The Green's functions for the homogeneous half-space II are given in [7]. For the nonhomogeneous half-space I, the dyadic Green's function $\bar{\bar{G}}{}^{\rm I}_F$ takes the form

$$\bar{\bar{G}}{}^{\rm I}_F = G^{yy}_{F_{\rm I}}\vec{e}_y\vec{e}_y + G^{zy}_{F_{\rm I}}\vec{e}_z\vec{e}_y \tag{8}$$

where the components $G^{yy}_{F_{\rm I}}$ and $G^{zy}_{F_{\rm I}}$, determined at this plane ($z = 0$), are found to be

$$G^{yy}_{F_{\rm I}} = \frac{\epsilon_1}{2\pi}\int_0^{+\infty} J_0(\lambda\rho)\frac{u_2 + \epsilon_r u_0 \tanh(u_2 H)}{u_2\,\text{DTM}}\lambda\, d\lambda \tag{9}$$

$$G^{zy}_{F_{\rm I}} = 0. \tag{10}$$

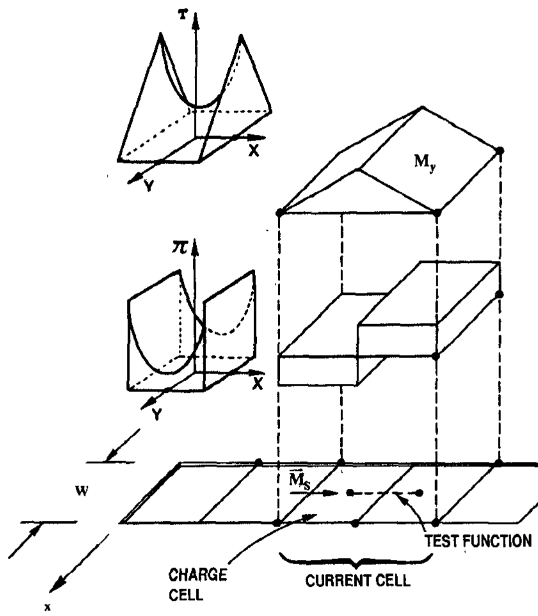

Fig. 3. Partial basis functions for equivalent magnetic current and charges.

The scalar magnetic Green's function $G^{\rm I}_U$ in half-space I is

$$G^{\rm I}_U = \frac{1}{2\pi\mu_0}\int_0^{+\infty} J_0(\lambda\rho)\frac{\lambda\, d\lambda}{\text{DTE}\cdot\text{DTM}} \times\left[\frac{\epsilon_r u_2}{\tanh(u_2 H)} + u_0 + (1-\epsilon_r)u_2\tanh(u_2 H) + \frac{\epsilon_r u_0}{u_2}\text{DTE}\tanh(u_2 H)\right]. \tag{11}$$

The various parameters involved in (9) and (11) are defined as

$$\text{DTM} = u_0\epsilon_r + u_2\tanh(u_2 H)$$

$$\text{DTE} = u_0 + u_2/\tanh(u_2 H)$$

$$u_0 = \left(\lambda^2 - k_0^2\right)^{1/2}$$

$$u_2 = \left(\lambda^2 - k_0^2\epsilon_r\right)^{1/2}$$

$$\rho = \left[(x_t - x_\nu)^2 + (y_t - y_\nu)^2\right]^{1/2}$$

and $J_0(\lambda\rho)$ is the Bessel function.

It is to be noticed that the zeros of DTE and DTM lead to TE and TM surface wave modes, respectively [8].

B. Electric Field Distributions

The moment method [9] is widely regarded as the numerical technique best suited for the resolution of integral equations encountered in the study of planar antennas [8], [10]. In this approach the slots are divided in the y direction into rectangular cells of equal size. The unknown densities (current and charge) are then expanded as a finite sum of basis functions as given in Fig. 3 [11].

It is to be noticed that the choice of the basis functions is determined by many factors, the main ones being the rela-

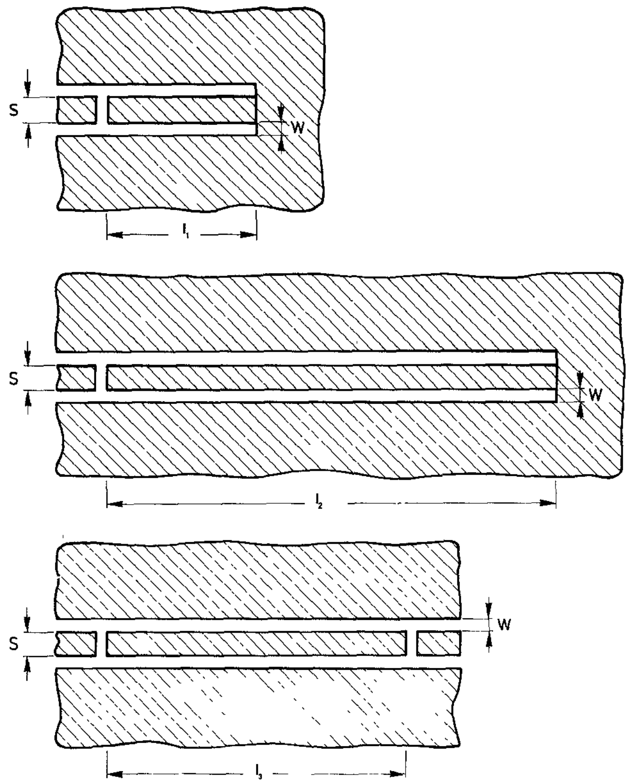

Fig. 4. Three series-gap-coupled straight resonators.

tively fast convergence of the solution and the ease of evaluating the matrix elements. For the case of slotline or coplanar waveguide the suitable basis functions were chosen to be of the form

$$T = \begin{cases} (1-|y|/a)/(1-(2x/W)^2)^{1/2} & \text{if } |y| < a \text{ and } |x| < W/2 \\ 0 & \text{elsewhere} \end{cases} \quad (12)$$

$$\pi = \begin{cases} 1/\left(1-(2x/W)^2\right)^{1/2} & \text{if } |y| < a \text{ and } |x| < W/2 \\ 0 & \text{elsewhere} \end{cases} \quad (13)$$

where a is the cell width

These basis function give the correct transverse variation of the current density n the slots taking into account fringing effects due to conductor thickness. The final inhomogeneous linear equations system is solved by matrix inversion.

Numerically the excitation of the CPW mode, usually called the odd mode due to the odd symmetry of the x electric field components with respect to the z axis, is simulated by two parallel current generators having equal and opposite phases. Each generator is placed at a distance of about a quarter of a wavelength from the line open circuit end and at a distance of three wavelengths from the studied short circuit.

The calculated fields in the two slots in zones far from both the generator and the discontinuity are of the form of standing waves of the fundamental propagating modes. Thus transmission line theory can be used to compute the reflection coefficient at a given reference plane; hence the equivalent impedance of a CPW short circuit end can be determined.

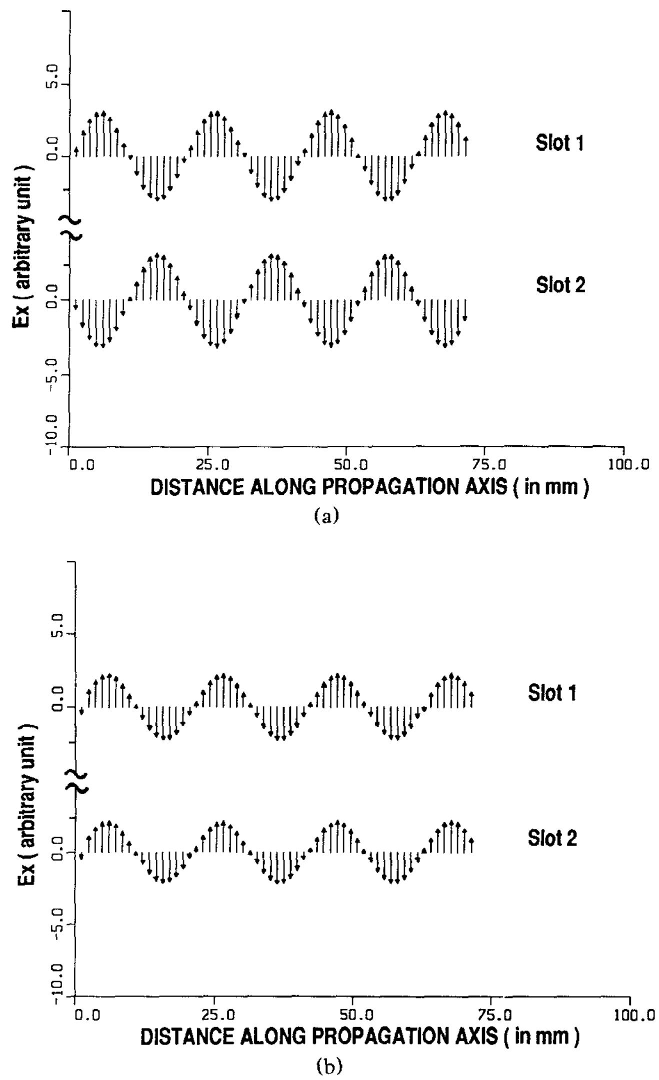

Fig. 5. (a) Electric fields distribution in the two slots of a short circuit CPW operating in the odd mode ($\epsilon_r = 2.2$, $H = 0.8$ mm, $W = 1.5$ mm, $S = 0.789$ mm). (b) Electric field distribution in the two slots of a short circuit CPW operating in the even mode ($\epsilon_r = 2.2$, $H = 0.8$ mm, $W = 1.5$ mm, $S = 0.789$ mm).

III. Measurement of CPW Short Circuit Reactance

The reactive part of the equivalent impedance of a pseudo CPW short circuit can be determined experimentally using the resonance method [12] of three series-gap-coupled straight resonators.

The straight CPW resonator is short-circuited from one end and is excited from the other end through a gap in the central conductor of the CPW (Fig. 4). The gap length is optimized to obtain a weak coupling between the resonator and its feeding line. Measuring the resonance frequencies for two straight resonators of lengths l_1, and l_2 ($l_2 \simeq 3l_1$) allows the determination of the line effective relative permittivity, ϵ_{eff}, and the combined end effects, Δl (short circuit end and gap discontinuity). At resonance one can write

$$l_i + \Delta l = \frac{(2n+1)c^*}{4f_{\text{res}}^{(i)}\sqrt{\epsilon_{\text{eff}}(f)}}, \qquad i = 1,2 \quad (14)$$

where n is an integer and c^* is the free-space light velocity.

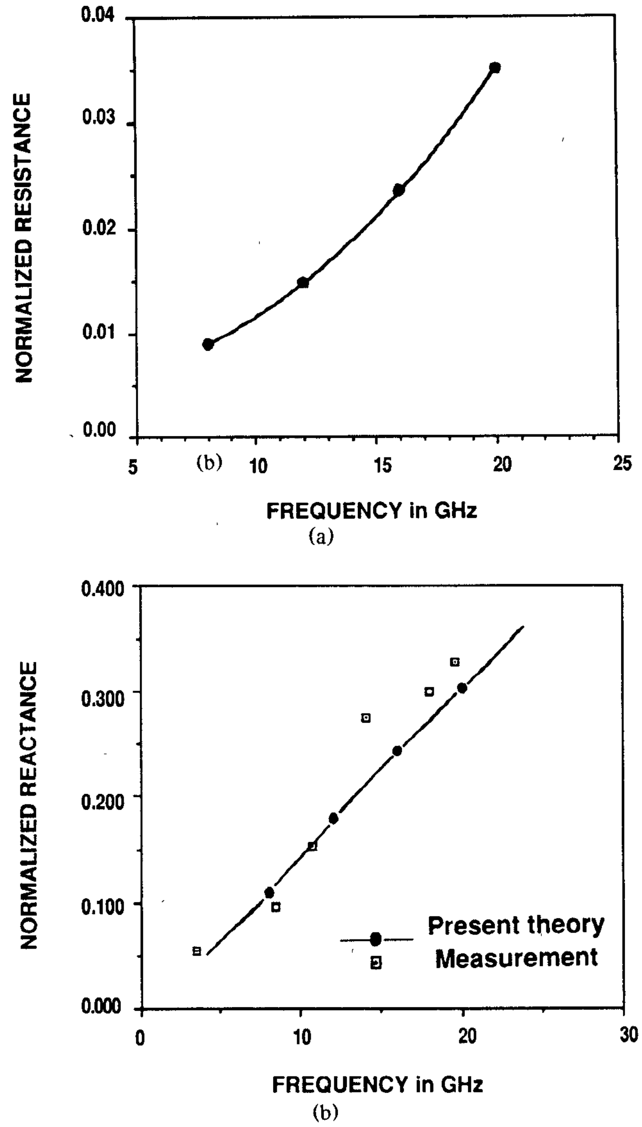

Fig. 6. Normalized end impedance of a shorted coplanar waveguide ($\epsilon_r = 2.2$, $H = 0.8$ mm, $W = 1.5$ mm, $S = 0.789$ mm).

In order to separate the two end effects, a third straight resonator, of length $l_3 = 2l_1$, loaded from its two ends by a gap identical to that used to feed the first two resonators is realized. The resonance condition can be written as

$$l_3 + 2\,\Delta l_{\text{gap}} = \frac{nc^*}{2f_{\text{res}}^{(3)}\sqrt{\epsilon_{\text{eff}}(f)}}. \tag{15}$$

In this way the short end and gap effects are separated and determined along with the CPW guided wavelength.

IV. Theoretical and Experimental Results

Two examples of calculated results that give the electric field distribution in the two slots are given in Fig. 5. The first one represents a CPW mode (odd mode), while the second represents an even coupled slots mode.

The normalized reactance obtained by the present theory is compared with our measurements in Fig. 6. The comparison shows a very good agreement.

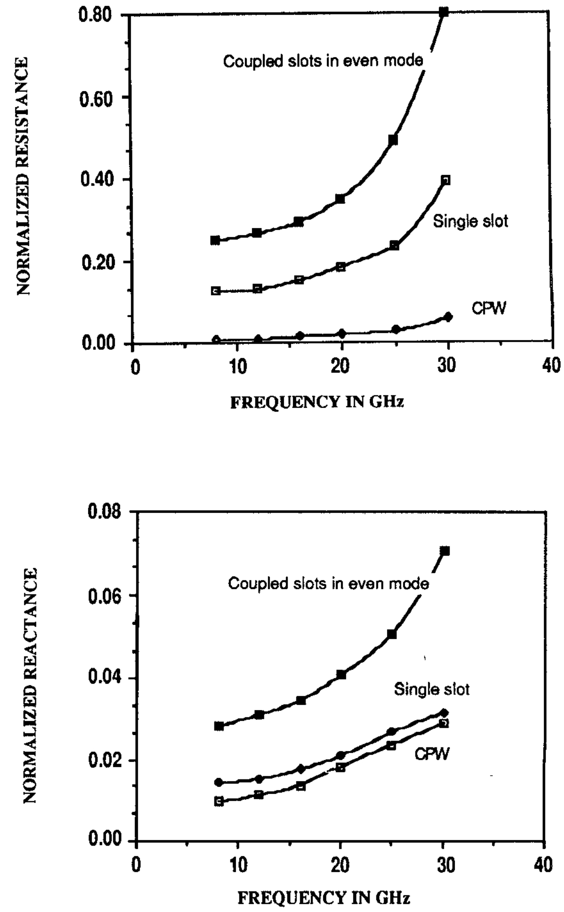

Fig. 7. End effects of a shorted end of a slotline and of coupled slots operating in the odd mode (coplanar mode) and the even mode ($\epsilon_r = 9.7$, $H = 0.635$ mm, $W/H = 0.4$, $S/H = 1$).

The present analysis has the advantage of being capable to determine also the resistive part of this short end of CPW due to free space and surface wave radiation. Fig. 6 gives an example of such results. It is seen that this resistance increases with frequency.

The results of a comparative study of the short-end equivalent impedance for a slotline and a CPW having the same slot width operating once in the odd mode (coplanar mode) and another time in the even mode are given in Fig. 7. The results show clearly that the CPW operating in odd mode has very low radiation properties when compared with the slotline and coupled slotlines operating in the even mode.

V. Conclusion

The integral equation technique solved by the moment method combined with the use of simple transmission line theory is shown to be a suitable and precise means for characterizing radiating discontinuities of CPW's. An experimental setup is shown to be capable of determining CPW short circuit end effects and of verifying our theoretical results.

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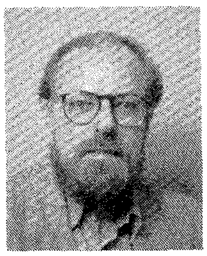

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